Steam Nozzles and Types

Nozzle is a duct by flowing through which the velocity of a fluid increases at the expense of pressure drop. If the fluid is steam, then the nozzle is called as Steam nozzle. The flow of steam through nozzles may be taken as adiabatic expansion. The steam possesses a very high velocity at the end of the expansion, and the enthalpy decreases as expansion occurs. Friction exists between the steam and the sides of the nozzle; heat is produced as the result of the resistance to the flow. The phenomenon of super saturation occurs in the steam flow through nozzles. This is because of the time lag in the condensation of the steam during the expansion.

The area of such duct having minimum cross-section is known as throat.

A fluid is called compressible if its density changes with the change in pressure brought about by the flow.

If the density changes very little or does not change, the fluid is said to be incompressible. Generally the gases and vapours are compressible, whereas liquids are incompressible.

Types of Nozzles:

There are three types of nozzles

1. Convergent nozzle
2. Divergent nozzle
3. Convergent-divergent nozzle.

Convergent Nozzle:

A typical convergent nozzle is shown in the Fig.1. In a convergent nozzle, the cross sectional area decreases continuously from its entrance to exit. It is used in a case where the back pressure is equal to or greater than the critical pressure ratio.

![Fig 1. Convergent nozzle](image-url)
**Divergent nozzle:**

The cross sectional area of divergent nozzle increases continuously from its entrance to exit. It is used in a case where the back pressure is less than the critical pressure ratio.

![Fig 2. Divergent nozzle](image)

**Convergent – Divergent nozzle:**

In this condition, the cross sectional area first decreases from its entrance to the throat and then again increases from throat to the exit. This case is used in the case where the back pressure is less than the critical pressure. Also, in present day application, it is widely used in many types of steam turbines.

![Fig 3. Convergent-Divergent nozzle](image)
Flow of steam Through Nozzle

Supersaturated flow or metastable flow of in Nozzles: As steam expands in the nozzle, the pressure and temperature in it drop, and it is likely that the steam start condensing when it strikes the saturation line. But this is not always the situation. Due to the high velocities, the time up to which the steam resides in the nozzle is small, and there may not be sufficient time for the needed heat transfer and the formation of liquid droplets due to condensation. As a result, the condensation of steam is delayed for a while. This phenomenon is known as super saturation, and the steam that remains in the wet region without holding any liquid is known as supersaturated steam. The locus of points where condensation occurs regardless of the initial temperature and pressure at the entrance of the nozzle is called the Wilson line. The Wilson line generally lies between 4 and 5 percent moisture curves in the saturation region on the h-s diagram in case of steam, and is often taken as 4 percent moisture line. The phenomenon of super saturation is shown on the h-s chart below:

Fig 4. The h-s diagram for the expansion of steam in the nozzle

Effects of Supersaturation:

The following are the effects of supersaturation in a nozzle.

1. The temperature at which the steam becomes supersaturated will be less than the saturation temperature corresponding to that pressure. Therefore, supersaturated steam will have the density more than that of equilibrium condition which results in the increase in the mass of steam discharged.
2. Supersaturation causesthe specific volume and entropy of the steam to increase.

3. Supersaturation reduces the heat drop. Thus the exit velocity of the steam is reduced.

4. Supersaturation increases the dryness fraction of the steam.

**Effect of Friction on Nozzles:**

1. Entropy is increased.

2. The energy available decreases.

3. Velocity of flow at the throat get decreased.

4. Volume of flowing steam is decreased.

5. Throat area required to discharge a given mass of steam is increased.

**Continuity and steady flow energy equations through a certain section of the nozzle:**

Where \( m \) denotes the mass flow rate, \( v \) is the specific volume of the steam, \( A \) is the area of cross-section and \( C \) is the velocity of the steam.

For steady flow of the steam through a certain apparatus, principle of conservation of energy states:

\[
h_1 + \frac{C_1^2}{2} + gz_1 + q = h_2 + \frac{C_2^2}{2} + gz_2 + w
\]

For nozzles, changes in potential energies are negligible, \( w = 0 \) and \( q = 0 \).

\[
h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}
\]

which is the expression for the steady state flow energy equation.
Things to remember

- Nozzle is a duct by flowing through which the velocity of a fluid increases at the expense of pressure drop. If the fluid is steam, then the nozzle is called as Steam nozzle.

- A fluid is said to be compressible if its density changes with the change in pressure brought about by the flow.

- If the density does not change or changes very little, the fluid is said to be incompressible. Usually the gases and vapors are compressible, whereas liquids are incompressible.

- There are three types of nozzles
  1. Convergent nozzle
  2. Divergent nozzle
  3. Convergent-divergent nozzle.

- **Effect of Friction on Nozzles:**
  1. Entropy is increased.
  2. The energy available decreases.
  3. Velocity of flow at the throat get decreased.
  4. Volume of flowing steam is decreased.
  5. Throat area required to discharge a given mass of steam is increased.
13.9 SUPERSATURATED OR METASTABLE FLOW THROUGH NOZZLE

The expansion of superheated steam from pressure $P_1$ to pressure $P_b$ can be represented by $AB$ as shown in Fig. 13.5. The change of phase should start at pressure $P_2$ where the expansion line meets the saturation line at point $C$. But condensation does not occur at point $C$, as the time available is very small (about 0.001s) due to high velocity of steam passing through the nozzle. Thus this phenomenon is delayed and the vapour continues to expand in dry condition even beyond the point $C$. This is represented by $CC_1$ and the condensation is suppressed. The vapour between the pressure $P_2$ and $P_3$ is said to be supersaturated or supercooled and this type of flow in nozzles is known as supersaturated or metastable flow of steam.

A limit to the supersaturated flow is fixed by Wilson line and the this line is the saturation line for all practical purposes. It is called supercooled flow because at any pressure between $P_2$ and $P_3$, the temperature of the vapour is always lower than the saturation temperature corresponding to that pressure. The difference of this temperature is known as degree of undercooling. When the expansion reaches to $C_1$, the condensation starts at constant enthalpy. This is represented by $C_1D$. $DE$ represents the normal adiabatic expansion up to the exit pressure.

13.10 EFFECTS OF SUPERSATURATION

The following are the effects of supersaturation in a nozzle.

(a) The temperature at which the supersaturation occurs will be less than the saturation temperature corresponding to that pressure. Therefore, the density of supersaturated steam will be more than that of equilibrium condition which gives the increase in the mass of steam discharged.

(b) Supersaturation increases the specific volume and entropy of the steam.

(c) Supersaturation reduces the heat drop. Thus the exit velocity of steam is reduced.

(d) Supersaturation increases the dryness fraction of the steam.
16.10 SUPER SATURATED FLOW (OR) METASTABLE FLOW THROUGH THE NOZZLE

The expansion of superheated steam from pressure $p_1$ to $p_2$ is shown in the Mollier diagram. The pressure $p_2$ is located in the wet region. Since the expansion is isentropic, the expansion through the nozzle is represented as a single vertical line from state 1 to state 2. The expansion process is always combined with condensation process when the steam is expanded beyond the saturation region. When the steam crosses the saturation line, theoretically, condensation should start. But in practical point of view, the flow of steam through the nozzle possess a great velocity may be in the range of supersonic velocity. Hence the time availability to start the process of condensation is very low. Hence the condensation does not occur at the point of intersection of saturation line and expansion line, point $x$. Let the pressure at point ‘$x$’ be $p_x$. The process of condensation is delayed and the vapour continues to expand in the dry state within the saturated region beyond point $x$. This region is represented between point $x$ and $y$. Practically, the condensation starts at point $y$. The condensation process is delayed (or) suppressed between $x$ and $y$. The vapour within the saturated region $xy$ is called super saturated or supercooled flow through the nozzle. For a super cooled steam, the temperature at any pressure is less than the saturation temperature at that pressure. It is important to have a limit for this supersaturated flow through the nozzle. The limit is characterised by a line called as wilson line. The wilson line is a saturation line and is approximately parallel to the theoretically correct saturation line. The wilson line passes though point $y$ as shown. Practically, this wilson line passes through 0.97 dryness fraction line. Beyond this wilson line, there is no super saturation. Once the steam crosses the wilson line condensation start, at constant enthalpy as shown between $yy’$ then the process of expansion

Fig. 16.7
A nozzle is a duct of smoothly varying cross-sectional area in which a steadily flowing fluid can be made to accelerate by a pressure drop along the duct.

**Applications**: Steam and Gas Turbines, Jet Engines, Rocket Motors, Flow Measurement.

**Assumption**: The flow of fluid is assumed to be one-dimensional and steady.

In one-dimensional flow it is assumed that the fluid velocity, and the fluid properties, change only in the direction of the flow. This means that the fluid velocity is assumed to remain constant at a mean value across the cross-section of the duct.

**Nozzle shape**

Consider a stream of fluid at pressure $p_1$, enthalpy $h_1$, and with a low velocity $V_1$. It is required to find the shape of duct which will cause the fluid to accelerate to a high velocity as the pressure falls along the duct.

**Assumptions**: The heat loss from the duct is negligibly small (i.e. adiabatic flow, $Q = 0$), and it is clear that no work is done on or by the fluid (i.e. $W = 0$).

Applying the energy equation between section 1 and any other section $X-X$ where the pressure is $p$, the enthalpy is $h$, and the velocity is $V$, we have

$$h_1 + \frac{V_1^2}{2} = h + \frac{V^2}{2}$$

i.e. $V^2 = 2(h_1 - h) + V_1^2$

or, $V = \sqrt{(2(h_1 - h) + V_1^2)}$  \hspace{1cm} (1)

If the area at section $X-X$ is $A$, and the specific volume is $v$, then,

Mass flow, $\dot{m} = \frac{VA}{v}$

Or, Area per unit mass flow, $\frac{A}{\dot{m}} = \frac{v}{V}$.  \hspace{1cm} (2)

Then substituting for the velocity $V$, from equation (1),

Area per unit mass flow = $\frac{v}{\sqrt{(2(h_1 - h) + V_1^2)}}$  \hspace{1cm} (3)

It can be seen from equation (3) that in order to find the way in which the area of the duct varies it is necessary to be able to evaluate the specific volume, $v$, and the enthalpy, $h$, at any section $X-X$. In order to do this, some information about the process undergone between section 1 and section $X-X$ must be known.

For the ideal frictionless case, since the flow is adiabatic and reversible, the process undergone is an isentropic process, and hence

$s_1 = (\text{entropy at section } X - X) = s$, say

Now using equation (2) and the fact that $s_1 = s$, it is possible to plot the variation of the cross-sectional area of the duct against the pressure along the duct. For a vapour this can be done using tables; for a perfect gas the procedure is simpler, since we have $pv^\gamma = \text{Constant}$, for an isentropic process. In either case, choosing fixed inlet conditions, then the variation in the area, $A$, the specific volume, $v$, and the velocity, $V$, can be plotted against the pressure along the duct. Typical curves are shown in Fig. 1. It can be seen that the area decreases initially, reaches a maximum, and then increases again.
This can be seen from equation (2),

\[ \text{Area per unit mass flow} = \frac{V}{V} \]

When \( v \) increases less rapidly than \( V \), then the area decreases; when \( v \) increases more rapidly than \( V \), then the area increases.

A nozzle, the area of which varies as in Fig. 1, is called a \textit{convergent-divergent} nozzle (Fig. 2). The section of minimum area is called \textit{throat} of the nozzle. It will be shown later that the velocity at the throat of a nozzle operating at its designed pressure ratio is the velocity of sound at the throat conditions. The flow upto the throat is \textit{subsonic}; the flow after the throat is \textit{supersonic}. It should be noted that a sonic or a supersonic flow requires a diverging duct to accelerate it.

The specific volume of a liquid is constant over a wide pressure range, and therefore the nozzles for liquids are always convergent, even at very high exit velocities (e.g. a fire-hose uses a convergent nozzle).

**Critical pressure ratio**

It is stated earlier that the velocity at the throat of a correctly designed nozzle is the velocity of sound. In the same way, for a nozzle that is convergent only, then the fluid will attain sonic velocity at exit if the pressure drop across the nozzle is large enough. The ratio of the pressure at the section where sonic velocity is attained to the inlet pressure of a nozzle is called the \textit{critical pressure ratio}.

Consider a convergent-divergent nozzle as shown in Fig. 3 and let the conditions at inlet and at any other section \( X-X \) be as shown in the figure.

In most practical applications the velocity at the inlet to a nozzle is negligibly small in comparison with the exit velocity. It can be seen from equation (2), \( \frac{A}{V} = \frac{v}{V} \), that a negligibly small velocity implies a very large area, and most nozzles are in fact shaped at inlet in such a way that the nozzle converges rapidly over the first fraction of its length; this is illustrated in the diagram of a nozzle inlet shown in Fig. 4.

Now from equation (1), neglecting \( V_1 \), we have

\[ V = \sqrt{2(h_i - h)} \]  

(4)
Since enthalpy is usually expressed in kilojoules per kilogram, then an additional constant of $10^3$ will appear within the root sign if $V$ is to be expressed in m/s. Then

Area per unit mass flow, $\frac{A}{m} = \frac{v}{V} = \frac{v}{\sqrt{(2(h_1 - h)}}$

For a perfect gas, it is possible to simplify the above equation by making use of the perfect gas laws. For a perfect gas, $h = c_p T$. Therefore,

Area per unit mass flow rate, $\frac{A}{m} = \frac{v}{V} = \frac{v}{\sqrt{2c_p(T_1 - T)}} = \frac{v}{\sqrt{2c_p T_1\left(1 - \frac{T}{T_1}\right)}}$

But $v = RT/p$, therefore,

Area per unit mass flow rate, $= \frac{\frac{RT}{p}}{\sqrt{2c_p T_1\left(1 - \frac{T}{T_1}\right)}}$

Let the pressure ratio, $p/p_1 = x$. Then for an isentropic process for a perfect gas:

$\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{1}{\gamma - 1}} = x^{\frac{1}{\gamma - 1}}$

Substituting for $p = xp_1$, $T = T_1x^{\frac{1}{\gamma - 1}}$, and $\frac{T}{T_1} = x^{\frac{1}{\gamma - 1}}$, we have

Area per unit mass flow rate, $= \frac{RT_1 x^{\frac{1}{\gamma - 1}}}{p_1 x^{\frac{1}{\gamma - 1}} \sqrt{2c_p T_1\left(1 - x^{\frac{1}{\gamma - 1}}\right)}}$

For fixed inlet conditions (i.e. $p_1$ and $T_1$ fixed), we have

Area per unit mass flow rate, $= \text{constant} \times \frac{x^{\frac{1}{\gamma - 1}}}{x^{\frac{1}{\gamma + 1}} \sqrt{\left(1 - x^{\frac{1}{\gamma - 1}}\right)}}$

$= \text{constant} \times \frac{1}{x^{\frac{1}{\gamma - 1}} \sqrt{\left(1 - x^{\frac{1}{\gamma - 1}}\right)}}$

$= \frac{\text{constant}}{x^{\frac{1}{\gamma}} \sqrt{\left(x^{\frac{2}{\gamma}} - x^{2\gamma} x^{\frac{1}{\gamma - 1}}\right)}}$

Therefore,
Area per unit mass flow rate \( = \frac{constant}{\sqrt{\left( x^{2/\gamma} - x^{(\gamma+1)/\gamma} \right)}} \) \( (5) \)

To find the value of the pressure ratio, \( x \), at which the area is a minimum it is necessary to differentiate equation (5) with respect to \( x \) and equate the result to zero. i.e. for minimum area

\[ \frac{d}{dx} \left\{ \frac{1}{(x^{2/\gamma} - x^{(\gamma+1)/\gamma})^{1/2}} \right\} = 0 \]

i.e. \[ \frac{2 x^{(2/\gamma)-1} - (\gamma + 1) x^{[(\gamma+1)/\gamma]-1}}{2(x^{2/\gamma} - x^{(\gamma+1)/\gamma})^{3/2}} = 0 \]

Hence the area is a minimum when

\[ x = \left( \frac{2}{\gamma + 1} \right)^{(\gamma+1)/\gamma} \]

\[ \text{CRITICAL PRESSURE RATIO} = \frac{p_c}{p_1} \] \( (6) \)

For AIR, \( \gamma = 1.4 \), therefore,

\[ \frac{p_c}{p_1} = \left( \frac{2}{1.4+1} \right)^{14/0.4} = 0.5283 \]

Hence for air at 10 bar, say, a convergent nozzle requires a back pressure of 5.283 bar, in order that the flow should be sonic at exit and for a correctly designed convergent-divergent nozzle with inlet pressure of 10 bar, the pressure at the throat is 5.283 bar.

For carbon dioxide, \( \gamma = 1.3 \), therefore,

\[ \frac{p_c}{p_1} = \left( \frac{2}{1.3+1} \right)^{13/0.3} = 0.5457 \]

Hence for carbon dioxide at 10 bar, a convergent nozzle requires a back pressure of 5.457 bar for sonic flow at exit, and the pressure at the throat of a convergent-divergent nozzle with inlet pressure 10 bar is 5.457 bar.

Similarly, the critical temperature ratio, \( \frac{T_c}{T_i} = \left( \frac{p_c}{p_1} \right)^{(\gamma-1)/\gamma} = \frac{2}{\gamma + 1} \) \( (7) \)

Equations (6) and (7) apply to perfect gases only, and not to vapours.

However, it is found that sufficiently close approximation is obtained for a steam nozzle if it is assumed that the expansion follows the law \( \rho v^k = \text{constant} \). The process is assumed to be isentropic and therefore the index \( k \) is an approximate index for steam. Usually,

\( k = 1.135 \) for steam initially\( \text{ dry saturated} \);

\( k = 1.3 \) \( \text{ for steam is initially superheated.} \)

Note that equation (7) cannot be used for a wet vapour undergoing an isentropic process.

For a PERFECT GAS, the critical velocity, \( V_c = \sqrt{\gamma RT_c} = \text{Velocity of sound, a} \) \( (8) \)

The critical velocity given by equation (8) is the velocity at the throat of a correctly designed convergent-divergent nozzle, or the velocity at the exit of a convergent nozzle when the pressure ratio across the nozzle is the critical pressure ratio.
1). Air at 8.6 bar and 190°C expands at the rate of 4.5 kg/s through a convergent-divergent nozzle into a space at 1.03 bar. Assuming that the inlet velocity is negligible, calculate the throat and the exit cross-sectional areas of the nozzle. Take $c_p$ for air = 1.005 kJ/kg K.

2). A fluid at 6.9 bar and 93°C enters a convergent nozzle with negligible velocity, and expands isentropically into a space at 3.6 bar. Calculate the mass flow per square meter of exit area:

(i) when the fluid is helium ($c_p = 5.19$ kJ/kg K);
(ii) when the fluid is ethane ($c_p = 1.88$ kJ/kg K).

Assume that both helium and ethane are perfect gases, and take the respective molar masses as 4 kg/kmol and 30 kg/kmol.

The steam nozzle

The properties of steam can be obtained from tables or from an $h-s$ chart, but in order to find the critical pressure ratio, and hence the critical velocity and the maximum mass flow rate, approximate formulae may be used. It is a good approximation to assume that steam follows an isentropic law $p v^k$ = constant, where $k$ is an isentropic index for steam (≠ a ratio of specific heats). As already mentioned,

$k = 1.135$ for steam initially dry saturated, and $\frac{p_c}{p_1} = 0.577$

$k = 1.3$ for steam initially superheated, and $\frac{p_c}{p_1} = 0.546$

The temperature at the throat, i.e. the critical temperature can be found from steam tables at the value of $p_c$ and $s_c = s_1$. The critical velocity can be found as before:

$V_c = \sqrt{\{2(h_c-h_1)\}}$

Where $h_c$ is read from tables or the $h-s$ chart at $p_c$ and $s_c$.

For isentropic flow, since $v dp = dh$ and $v p^{1/k} = \text{constant}$, we can write between any two states 1 and 2:

$h_1-h_2 = \frac{2}{1} v dp = \frac{-v p^{1/k}}{-(1/k)+1} \left\{p_2^{-(1/k)+1} - p_1^{-(1/k)+1}\right\}$

i.e. $h_1-h_2 = \frac{k}{k-1} \left\{p_1 v_1 - p_2 v_2\right\} = \frac{V_2^2 - V_1^2}{2}$ (9)

Supersaturation

When a superheated vapour expands isentropically, condensation within the vapour begins to form when the saturated vapour line is reached. As the expansion continues below this linen into the wet region, then condensation proceeds gradually and the dryness fraction of the steam becomes progressively smaller. This is illustrated on $T-s$ and $h-s$ diagrams in Figs. 5(a) and 5(b). Point A represents the point at which condensation within the vapour just begins.
It is found that the expansion through the nozzle is so quick that condensation within the vapour does not occur. The vapour expands as a superheated vapour until some point at which condensation occurs suddenly and irreversibly. The point at which condensation occurs may be within the nozzle or after the vapour leaves the nozzle.

Up to the point at which condensation occurs the state of the steam is not one of stable equilibrium, yet it is not one of unstable equilibrium, since a small disturbance will not cause condensation to commence. The steam in this condition is said to be in a metastable state; the introduction of a large object (e.g. measuring instrument) will cause condensation to occur immediately.

Such an expansion is called a supersaturation expansion.

Assuming isentropic flow, as before, a supersaturation expansion in a nozzle is represented on a $T-s$ and $h-s$ diagrams in Figs. 6(a) and 6(b) respectively. Line 1-2 on both diagrams represents the expansion with equilibrium throughout the expansion. Line 1-R represents supersaturated expansion. In supersaturated expansion the vapour expands as if the vapour line did not exist, so that line 1-R intersects the pressure line $p_2$ produced from the superheat region (shown chain-dotted). It can be seen from Fig. 6(a) that the temperature of the supersaturated vapour at $p_2$ is $t_R$, which is less than the saturation temperature $t_2$, corresponding to $p_2$. The vapour is said to be supercooled and the degree of supercooling is given by $(t_2 - t_R)$. Sometimes a degree of supersaturation is defined as the ratio of the actual pressure $p_2$ to the saturation pressure corresponding to the temperature $t_R$.

It can be seen from Fig.6(b) that the enthalpy drop in supersaturated flow $h_1 - h_R$ is less than the enthalpy drop under equilibrium conditions. Since the velocity at exit, $V_2 = \sqrt{2(h_1 - h_2)}$, it follows that the exit velocity for supersaturated flow is less than that for equilibrium flow. Nevertheless, the difference in enthalpy drop is so small, and since the square root of the enthalpy drop is used for finding $V_2$, then the effect on exit velocity is small.

If the approximations for isentropic flow are applied to the equilibrium expansion, then for the process illustrated in Figs. 6(a) and 6(b), the expansion from 1 to A obeys the law $pV^{1.3} = \text{constant}$, and the expansion
from A to 2 obeys the law $p v^{1.135} = \text{constant}$. The equilibrium expansion and the supersaturated expansion are shown on a $p-v$ diagram in Fig. 7, using the same symbols as in Fig. 6. It can be seen from Fig. 7 that the specific volume at exit with supersaturated flow, $v_R$, is considerably less than the specific volume at exit with equilibrium flow, $v_2$. Now the mass flow through a given exit area, $A_2$, is given for equilibrium flow

$$n_k = \frac{A_2 V_2}{v_2}$$

And for supersaturated flow

$$n_k = \frac{A_2 V_R}{v_R}.$$  

It has been pointed out that $V_2$ and $V_R$ are nearly equal; therefore, since $v_R < v_2$, it follows that the mass flow with supersaturated flow is greater than the mass flow with equilibrium flow. It was this fact, proved experimentally that led to the discovery of the phenomenon of supersaturation.

**Problem:**

A convergent-divergent nozzle receives steam at 7 bar and 200°C and expands it isentropically into a space at 3 bar. Neglecting the inlet velocity, calculate the exit area required for a mass flow of 0.1 kg/s:

(i) when the flow is in equilibrium throughout;

(ii) when the flow is supersaturated with $p v^{1.3} = \text{constant}$.
STEAM TURBINE

Classification same as explained earlier for hydraulic turbines.

Impulse Turbine

The most basic turbine takes a high-pressure, high-enthalpy fluid, expands it in a fixed nozzle, and then uses the rate of change of angular momentum of the fluid in a rotating passage to provide the torque on the rotor. Such a machine is called an impulse turbine. A simple example of an impulse turbine is shown in Figs. 8 (a) and 8(b). Since the fluid flows through the wheel at a fixed mean radius, then the change of linear momentum tangential to the wheel gives a tangential force that causes the wheel to rotate. Assume initially that the fluid is able to enter and leave the wheel passages in the tangential direction with an absolute velocity at inlet $V_i$, and an absolute velocity at exit, $V_e$, as shown in Fig. 9; the blade velocity is denoted by $U$.

The rate of increase of fluid momentum in the tangential direction from left to right in Fig. 9 gives the tangential force acting on the fluid, i.e. Force on the fluid from left to right $= n\Delta (V_e - V_i)$ (assuming a constant mass flow rate, $n\Delta$).

An equal and opposite force, $F$, must act on the blades, i.e. $F = n\Delta (V_i + V_e)$ from left to right.

The torque acting on the wheel is then given by

$T = n\Delta R (V_i + V_e)$

Where $R$ is the radius of the wheel, and the rate at which work is done for a rotational speed of $N$ is

$W = 2\pi N T = 2\pi N n\Delta R (V_i + V_e) = n\Delta U (V_i + V_e)$

Where $U$ is the blade tangential speed $= 2\pi N R$.

Referring to the Fig. 10, the velocity of the fluid relative to the blade at inlet is $W_i = (V_i - U)$ and the velocity of the fluid relative to the blades at outlet in the direction of the blade movement is $W_e = (-V_e - U)$. In the absence of friction the relative velocity at inlet is equal in magnitude to the relative velocity at outlet, i.e. $V_i - U = (-V_e - U)$.
\[ V_e = V_i - 2U \]

Substituting for \( V_e \) in the previous equation for \( \mathbf{W^e} \), we have
\[ \mathbf{W^e} = nB\mathbf{U}(V_i + V_i - 2U) \]
\[ = 2nB\mathbf{U}(V_i - U) \]

Figure 11.5: Absolute and relative velocities for a simple impulse turbine blade

Figure 11.6: Inlet (a) and (b) outlet blade velocity diagrams for an impulse turbine and a composite diagram (c)

Figure 11.7: Absolute velocities at inlet and exit and the forces produced
Problem: The velocity of steam leaving the nozzles of an impulse turbine is 900 m/s and the nozzle angle is 20°. The blade velocity is 300 m/s and the blade velocity coefficient is 0.7. Calculate for a mass flow of 1 kg/s, and symmetrical blading:

(i) the blade inlet angle;
(ii) the driving force on the wheel;
(iii) the axial thrust;
(iv) the diagram power, and
(v) the diagram efficiency.

[29°24'; 927.7 N per kg/s; 92.3 N per kg/s; 278.3 kW; 68.7%]

Figure 11.9: Diagram efficiency against blade speed ratio for a single-stage impulse turbine
Figure 11.10: Pressure-compounded impulse turbine showing pressure and velocity variations
Figure 11.11: Two row velocity compounded impulse turbine showing pressure and velocity variations

Figure 11.12: Velocity diagrams for a two-row velocity-compounded impulse turbine
Figure 11.13: Diagram efficiency against blade speed ratio for a two-row velocity-compounded impulse turbine

Figure 11.14: Pressure compounded two-row velocity-compounded impulse turbine showing pressure and velocity variations
Problem: The first stage of a turbine is a two-row velocity-compounded impulse wheel. The steam velocity at inlet is 600 m/s, the mean blade velocity is 120 m/s, and the blade velocity coefficient for all blades is 0.9. The nozzle angle is 16° and the exit angles for the first row of moving blades, the fixed blades, and the second row of moving blades, are 18, 21, and 35° respectively. Calculate:

(i) the blade inlet angles for each row;
(ii) the driving force for each row of moving blades and the axial thrust on the wheel, for a mass flow rate of 1 kg/s;
(iii) the diagram power per kg per second steam flow, and the diagram efficiency for the wheel;
(iv) the maximum possible diagram efficiency for the given steam inlet velocity and nozzle angle.

Figure 11.16: Diagram showing blade passage width (a) and length (b) for impulse blading
Problem: For the nozzles and wheel of Example the steam flow is 5 kg/s and the nozzle height is 25 mm. The specific volume of the steam leaving the nozzles is 0.375 kg/m$^3$. Neglecting the wall thickness between the nozzles, and assuming that all blades have a pitch of 25 mm and exit tip thickness of 0.5 mm, calculate:

(i) the length of the nozzle arc;
(ii) the blade height at exit from each row.

\[0.454 \text{ m}; l_1 = 0.0327 \text{ m}, l_1 = 41.5 \text{ mm}, l_2 = 44.2 \text{ mm}\]

Applying the steady flow energy equation to the fixed blades

\[h_0 - h_1 = \frac{V_i^2 - V_e^2}{2}\]

(This assumes that the velocity of steam entering the fixed blade is equal to the absolute velocity of the steam leaving the previous moving row; it therefore applies to a stage which is not the first).

Similarly for the moving blades,

\[h_1 - h_2 = \frac{W_e^2 - W_i^2}{2}\]

From Figure……, \(W_e = V_i\) and \(W_i = V_e\), therefore,

\[h_0 - h_1 = h_1 - h_2 \quad \text{or} \quad h_0 = 2 h_1 - h_2\]

i.e. \(h_0 - h_2 = 2(h_1 - h_2)\)

Therefore for this case, the DEGREE OF REACTION

\[\Lambda = \frac{h_1 - h_2}{h_0 - h_2} = \frac{1}{2}\]

This type of blading is called the PARSON’S HALF DEGREE REACTION or 50% REACTION TYPE. The energy input to the moving blade wheel can be written as

\[\frac{V_i^2}{2} + \frac{W_e^2 - W_i^2}{2}\]

Therefore, since \(W_e = V_i\), this becomes
\[ V_i^2 - \frac{W_i^2}{2} \]

From the velocity triangle:
\[ W_i^2 = V_i^2 + U^2 - 2V_iU \cos \alpha_i \]

I.e. Energy input \( = V_i^2 - \left( \frac{V_i^2 + U^2 - V_iU \cos \alpha_i}{2} \right) \)

\[ = V_i^2 - U^2 + 2V_iU \cos \alpha_i \]

Rate of doing work per unit mass flow rate = \( U \Delta V_\theta \)

Also \( \Delta V_\theta = ED = 2V_i \cos \alpha_i - U \), therefore

Rate of doing work per unit mass flow rate = \( U(2V_i \cos \alpha_i - U) \)

Therefore the diagram efficiency of the 50% reaction turbine is given by

\[ \eta_d = \frac{\text{Rate of doing work}}{\text{energy input}} \]

i.e.
\[ \eta_d = \frac{2U(2V_i \cos \alpha_i - U)}{V_i^2 - U^2 + 2V_iU \cos \alpha_i} = \frac{2(U/V_i)(2 \cos \alpha_i - U/V_i)}{1 - (U/V_i)^2 + 2(U/V_i) \cos \alpha_i} \]

where \( (U/V_i) \) is the blade speed ratio.

By equating \( d\eta_d / d(U/V_i) \) to ZERO, the value of blade speed ratio for maximum diagram efficiency can be shown to be given by:
\[ U/V_i = \cos \alpha_i \]

Rate of doing work = \( U(2V_i \cos \alpha_i - U) = U \left( 2V_i \frac{U}{V_i} - U \right) = U^2 \) for maximum diagram efficiency

Substituting \( U/V_i = \cos \alpha_i \) in the expression for diagram efficiency, we get

Maximum diagram efficiency = \( \frac{2 \cos^2 \alpha_i}{1 + \cos^2 \alpha_i} \)

For the optimum blade speed ratio a blade velocity diagram as shown in Fig….. is obtained (i.e. \( U = V_i \cos \alpha_i \))

The variation of \( \eta_d \) with blade speed ratio for the simple impulse turbine and the reaction stage are shown in Fig….. It can be seen that for the reaction turbine the curve is reasonably flat in the region of the maximum value of diagram efficiency, so that a variation in \( \cos \alpha_i \), and hence \( U/V_i \), can be accepted without much variation in the diagram efficiency from the maximum value.

The variation of pressure and velocity through a reaction turbine is shown in Fig….. The pressure falls continuously as the steam passes over the fixed and moving blades of each stage. The steam velocities are low compared with those of the impulse turbine, and it can be seen from the diagram that the steam velocity is increased in each set of fixed blades. It is no longer convenient to talk of “nozzles” and “blades”, since in the reaction turbine both fixed and moving blades act as nozzles. It is usual to refer to the two sets of blades as the stator blades and the rotor blades.

The pressure drop across the rotor produces an end thrust equal to the product of the pressure difference and the area of the annulus in contact with the steam. For the 50% reaction turbine the thrust due to the change in axial velocity is zero, but the side thrust is nevertheless greater than that of an equivalent impulse turbine, and larger thrust bearings are fitted. The net end thrust can be reduced by admitting the steam to the casing at the mid-section and allowing it to expand outwards to each end of the casing, passing over identical sets
of blades. This has the additional feature of reducing the blade height at a given wheel for a given total mass flow of steam.

**Losses in Turbine**

The losses which are of interest thermodynamically are the internal losses incurred as the fluid passes through the blades. The losses may be classified in one of the two groups:

(i) friction losses  
(ii) leakage losses  

Group (i) indicates friction losses in the nozzles, in the blades, and at the discs which rotate in the fluid. Group (ii) includes losses at admission to the stages and leakage at glands and seals, and the residual velocity loss.
Overall efficiency, stage efficiency, and reheat factor

Overall efficiency:
It has been shown that as a fluid expands through a turbine, there are friction effects between the fluid and the enclosing boundary surfaces of the nozzles and blade passages. Further losses are produced by leakage. Both of these are irreversibilities in the expansion process and there is a reduction in the useful enthalpy drop in the case of a turbine. Refer to the Fig…….

The overall isentropic efficiency of a turbine is defined as

\[ \eta_0 = \frac{h_1 - h_2}{h_1 - h_{2,s}} = \frac{\Delta h}{\Delta h_i} \]

The overall efficiency so defined depends only on the change of properties of the fluid during the expansion.

Stage efficiency and reheat factor

The expansion of the fluid through the successive stages of a reaction turbine can be represented on an h-s diagram as shown in Fig……. The procedure followed above for the whole turbine can be applied to each stage separately, and the dotted line joins the points representing the state of the steam between each stage. The dotted line is called the condition curve, although it does not give a continuous state path since in between the known points the processes are irreversible.

Considering any one stage, the available enthalpy drop of the stage can be represented by \( \Delta h_i \), wher subscript I refers to any stage from 1 to n, and the isentropic enthalpy drop between the same pressures can be represented by \( \Delta h_{i,s} \). Then a stage efficiency can be defined as

\[ \eta_s = \frac{\Delta h_i}{\Delta h_{i,s}} \]

From an inspection of Fig……., it is seen that \( BC < \Delta h_{i,2} \), etc. since the lines of constant pressure diverge from left to right on the diagram.

\[ \sum \Delta h_i > AB + BC + ..... + MN \]

i.e. \( \sum h_{i,s} > \Delta h_{i,0} \)

But \( \Delta h_i = \eta_s \Delta h_{i,s} \), and if it can be assumed that the stage efficiency is the same for each stage, then

\[ \sum_{i=1}^{n} \Delta h_i = \eta_s \sum_{i=1}^{n} h_{i,s} \]
Therefore
\[ \Delta h_0 = \eta_s \sum_{i=1}^{n} h_{si} \]
Dividing by \( \Delta h_{s0} \), we have
\[ \frac{\Delta h_0}{\Delta h_{s0}} = \eta_s \frac{\sum_{i=1}^{n} h_{si}}{\Delta h_{s0}} \]
Or \( \eta_0 = \eta_s \times (R.F.) \)
Where RF is the **Reheat Factor**

i.e. \( \text{R.F.} = \frac{\sum_{i=1}^{n} h_{si}}{\Delta h_{s0}} = \frac{\eta_0}{\eta_s} \)

Since \( \sum_{i=1}^{n} h_{si} \) is always greater than \( \Delta h_{s0} \), it follows that R.F. is always greater than unity; R.F. is usually of the order of 1.04 for a steam turbine.

**Problem:**

(i) Steam at 15 bar and 350°C is expanded through a 50% reaction turbine to a pressure of 0.14 bar. The stage efficiency is 75% for each stage, and the R.F. is 1.04. The expansion is to be carried out in 20 stages and the diagram power is required to be 12 000 kW. Calculate the flow of steam required, assuming that the stages all develop equal work.

(ii) In the turbine above at one stage the pressure is 1 bar and the steam is dry saturated. The exit angle of the blades is 20°, and the blade speed ratio is 0.7. If the blade height is one-twelfth of the blade mean diameter, calculate the value of the mean blade diameter and the rotor speed.

   [Answer: 64 770 kg/h; 1.3 m, 2067 rev/min]

**Problem:**

In a reaction stage of a steam turbine the nozzle angle is 20°, and the absolute velocity of the steam at inlet to the moving blades is 240 m/s. The blade velocity is 210 m/s. If the blading is designed for 50% reaction, determine,

(i) the blade angle at inlet and exit

(ii) the enthalpy drop per unit mass of steam in the moving blades and in the complete stage

(iii) the diagram power for a steam flow of 1 kg/s, and

(iv) the diagram efficiency.